MATH 312
Lecture 1 : Appraisal.

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BOOK Strang, Intro to Linear Algebra (4th ed)

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~~(Libgen)~~

- NAME
- YEAR
- MAJOR
- PREV LIN ALG Y/N

- Matlab / Maple ?
- office Hrs. Pref MW (after class)
- "Interests" ?
  - Bio
  - Computers
  - Pysics, engineering ?

$A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Q

a) Find $\vec{v} \cdot \vec{v}$ and $\|\vec{v}\|$.

b) Find A^{-1}

c) Solve for x & y in $A\begin{bmatrix} x \\ y \end{bmatrix} = \vec{v}$

⟸ If you can't do these, you WILL fall behind.

(2.1 – 2.3)

I. Algebra and Geometry in Equations.

We start with trying to solve for x and y;
but in three "different" cases:

a $\boxed{\begin{aligned} 4x + 7y &= 1 \\ x - 2y &= 4 \end{aligned}}$

b $\boxed{\begin{aligned} 4x + 7y &= 1 \\ y &= -1 \end{aligned}}$

c $\boxed{\begin{aligned} x &= 2 \\ y &= -1 \end{aligned}}$

EASIER!!

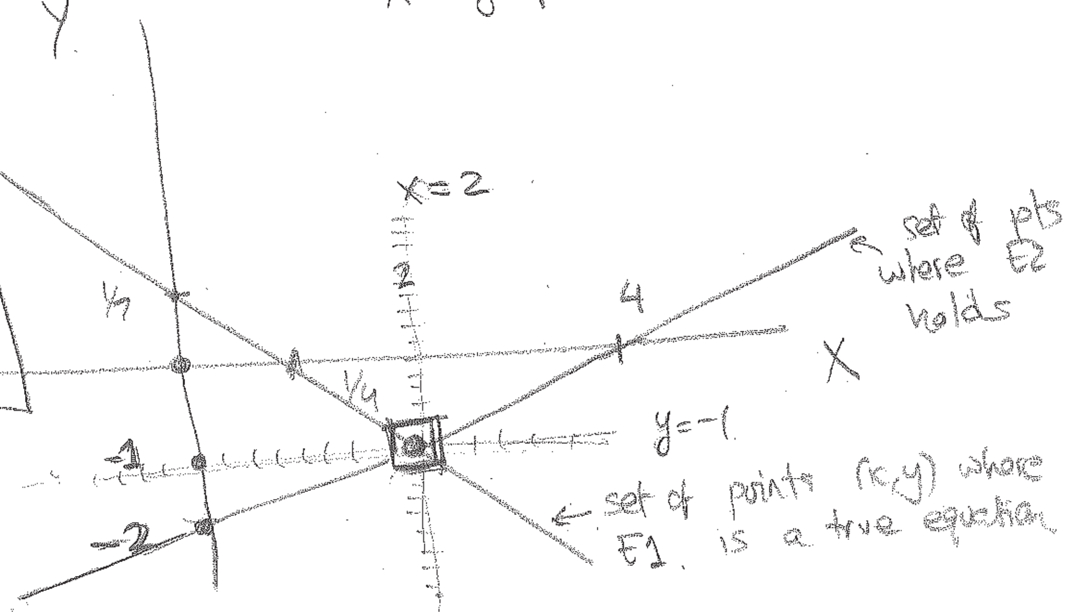
a) Solve it
b) Solve it
c) Ha!

Principle: if $\langle x \rangle = \langle y \rangle$ and we do the same thing to x that we do to y, then equality is preserved.

Return to $\quad 4x + 7y = 1 \rightarrow E1$
$\qquad\qquad\quad x - 2y = 4 \rightarrow E2$

A graphical solution!

Principle: Look for intersections!



II Gaussian Elimination: A systematic approach

Again: $\quad \boxed{4}x + 7y = 1 \qquad E1$ (Pivot)
$\qquad\qquad x - 2y = 4 \qquad E2$

We try to eliminate E2's dependence on "x" by using the "pivot" as a DIVISOR:

$$-\frac{4x}{4} - \frac{7y}{4} = -\frac{4}{4}$$
$$= \cancel{-x} - \frac{7}{4}y = -1$$
$$+ \quad \cancel{x} - 2y = 4$$
$$\left(-\left(\frac{7}{4}\right)+2\right) = 3$$
$$y\left(\frac{-7+8}{4}\right) = 3$$

$$E2' = E2 - \tfrac{1}{4}E1$$

get: $\quad 0 + y = -1$.

$$\cancel{x} - 2y = 4$$
$$-\cancel{x} - 7/4\,y = -1/4$$
$$-15/4\,y = +15/4$$
$$y = -1$$

New System:

$$4x + 7y = 1$$
$$y = -1$$

Look Ma, No x! ⟶

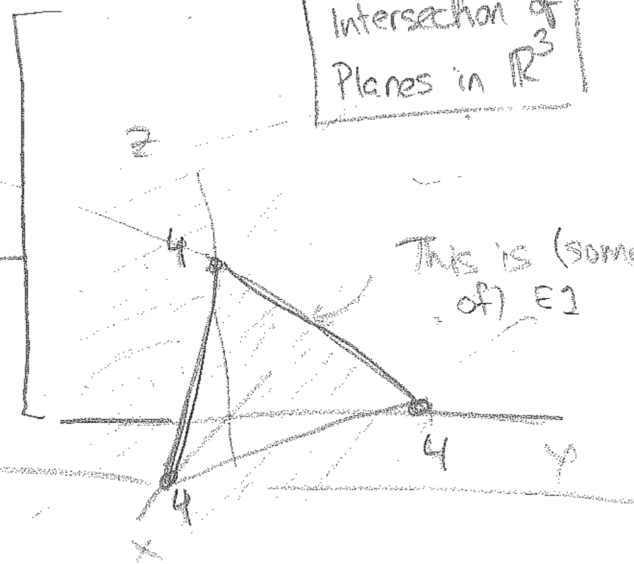
Now, "back-sub" $-1 = y$ into E1:

$$4x - 7 = 1$$
$$x = 2\ !$$

Let's do a 3x3 system for street-cred:

Intersection of Planes in $\mathbb{R}^3$

$$\begin{aligned} E1 &\quad x + y + z = 4 \\ E2 &\quad x - 2y - 2z = 7 \\ E3 &\quad 2x - 3y + 3z = -5 \end{aligned}$$



Already, geometry is hard!
BUT Algebra still works ☺

"ALGORITHM" for Gaussian Elimination.

$n = 1$

- Rearrange your equations so that the n-th variable in the n-th equation has a NON-ZERO coefficient → (called "PIVOT")

  (our system is okay!)

- Use this NON-ZERO coefficient to zero-out all the stuff BELOW it.

- Repeat with $n \leftarrow n+1$ until system is TRIANGULAR, i.e.

iterate

All 0's here →

Iteration 1
Pivot is "1"
$E1 = \underline{\underline{1x}} + y + z = 4$

$$\begin{aligned} E2' &= E2 - E1 \Rightarrow 0 - 3y - 3z = 3 \\ E3' &= E3 - 2E1 \Rightarrow 0 - 5y + z = -13 \end{aligned}$$

New system:

$$\begin{aligned} x + y + z &= 4 \quad \text{—— } E1 \\ -3y - 3z &= 3 \quad \text{—— } E2' \\ -5y + z &= -13 \quad \text{—— } E3' \end{aligned}$$

Iteration 2
Pivot is "-3"
$E2' = \underline{\underline{-3y}} - 3z = 3$

$$E3'' = E3' - \tfrac{5}{3}E2' \Rightarrow 6z = -18$$

So, we have our triangular system:

$$\begin{aligned} x + y + z &= 4 \qquad & E1 \\ -3y - 3z &= 3 \qquad & E2' \\ 6z &= -18 \qquad & E3'' \end{aligned}$$

This is EASY to solve BACKWARDS:

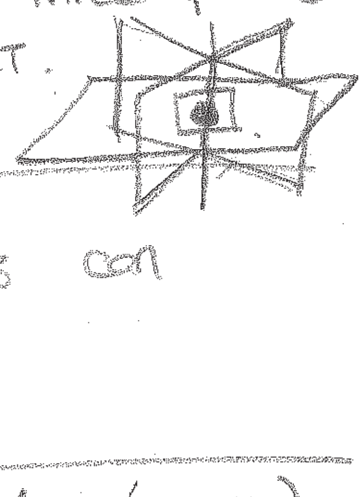
FIRST, use $E3''$ to get $z = -3$.

SECOND, use this z-value in $E2'$ to get $y = 2$.

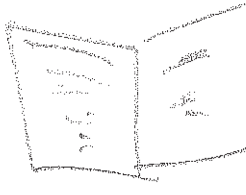
THIRD, use BOTH y and z-values in $E1$ to get $x = 5$.

So, the final answer is $\boxed{\begin{aligned} x &= 5 \\ y &= 2 \\ z &= -3 \end{aligned}}$.

This is the ONLY point where the three planes (arising from E1, E2 and E3) INTERSECT.



On FRIDAY, we will see how things can go WRONG with our algorithm.

READ:  Strang: chapter 1 (easy).
Chapter 2, Sections 1, 2, 3.